Abstract—In this letter, we propose two-way amplify-and-forward relaying in conjunction with adaptive modulation in order to improve spectral efficiency of relayed communication systems while monitoring the required error performance. We also consider a multiple relay network where only the best relay node is utilized so that the diversity order increases while maintaining a low complexity of implementation as the number of relays increases. Based on the best relay selection criterion, we offer an upper bound on the signal-to-noise ratio to keep the performance analysis tractable. Our numerical examples show that the proposed system offers a considerable gain in spectral efficiency while satisfying the error rate requirements.

Index Terms—Two-way relaying, amplify-and-forward relaying, best relay selection and adaptive modulation.

I. INTRODUCTION

In order to prevent a loss of the spectral efficiency by half-duplex transmission, Laneman et al. proposed in [1] an adaptive relaying strategy named incremental relaying which reduces the loss in spectral efficiency by deciding on the usage of the relay node based on the feedback channel information. On the other hand, two-way relaying (TWR) protocol was proposed and presented in [2]–[4] to also overcome the loss in the spectral efficiency due to half-duplex transmission. Specifically, both the two user terminals send their signals to the relay node at the same time (during Phase I), and then the relay node broadcasts its normalized signal to both the two user terminals (during Phase II). Thus, there exist two kinds of relaying paths, and the overall spectral efficiency can increase. Some additional works have been published on this topic. For instance, authors in [5] derived the achievable rate for TWR channel, and authors in [6] showed the achievable spectral efficiency of TWR based on decode-and-forward (DF) and amplify-and-forward (AF) relaying protocol, respectively and compared their performances with one-way relaying (OWR). While the authors in [7] provided a bound of the spectral efficiency of TWR based on DF protocol, the authors in [8] derived upper and lower bounds of the spectral efficiency of TWR based on AF protocol and extended the work to the case where both source and destination nodes have two antennas and the relay has only one antenna. In the multiple relay nodes environment, authors in [9] applied the relay selection to TWR and provides its statistic analysis. For another previous work dealing with the relay selection, in [10], the authors showed the performance bounds of the spectral efficiency of TWR based on AF protocol over the multiple relays network in which only the best relay is performed.

The achievable spectral efficiency addressed in the above literatures is considered in terms of the Shannon’s channel capacity which is an ideal performance bound, that can be used as a bench-mark for practical schemes. In order to obtain a more realistic spectral efficiency, we propose in this paper TWR with adaptive modulation (TWR-AM) based on AF relaying in which discrete rates [11], [12] are applied to each relaying path of TWR. Moreover, we consider a multiple relay environment where only the best relay node is used in order to keep the diversity gain and maintain a low complexity. From a practical implementation standpoint, we propose the best relay selection criterion as a simple solution and show that its performance is adequate in comparison with the optimal scheme.

Based on the provided effective SNR and the selection criterion, we derive the performance analysis of TWR-AM in terms of average spectral efficiency and average bit error rate (BER). We also show that TWR-AM with half-duplex transmission can provide considerable gains in spectral efficiency in comparison with (i) OWR-AM transmission with half-duplex and (ii) direct path transmission with adaptive modulation (D-AM) and full-duplex transmission. For the system configuration, we consider non-identically, independent distributed (non-i.i.d.) Rayleigh fading channels over the network. However, when we consider i.i.d. channels per hop, the path loss is taken into account [13] and a linear relaying model [6] is considered for a fair comparison.

II. TWR-AM

A. System Description

Fig. 1 depicts a two-way linear relaying network with two users and one relay node in which normalized distances are utilized as $d_{1,2} = d_{1,r}$ (user 1-relay) + $d_{r,2}$ (relay-user 2), and their corresponding channels between node $A$ and node

\[1\] Our goal is to achieve a considerable gain in spectral efficiency while satisfying a certain error performance.
for the relay paths per hop, we assume that all the distances also applied to the selected relay node. In the i.i.d. assumption is utilized to make a communication link. The AF protocol is a relay candidate which has the best end-to-end performance as depicted in Fig. 2. In this proposed scheme, only one relay can be chosen in a similar manner as opportunistic relaying which uses a short duration flag packet [14], but has a different policy for the selection criterion. Since there are two different paths in TWR, the balance of the two hops is more important than for the conventional one-way relaying. For example, if one channel gain of the two hops is very high, one path will have a good link gain, but the other path will suffer from a very poor link gain due to the high interference. Therefore, the best relay selection criterion should be different from the conventional opportunistic relaying. In addition, the spectral efficiency of the proposed best relay selection scheme may be enhanced by considering all possible cases. However, in such parameter case, a certain centralized system is required over the multiple relays network and all the channel state information should be synthesized to consider all possibilities on the system. In order to simplify the practical implementation constraints, we propose the best relay selection criterion in (4) which comes from the effective SNR in the channel capacity and choose the best relay which maximizes the proposed selection criterion.

$$C_i = \frac{(\alpha_{1,r_i} \alpha_{2,r_i})^2}{(2\alpha_{2,r_i} + \alpha_{1,r_i} + 1)(2\alpha_{1,r_i} + \alpha_{2,r_i} + 1)}$$

In (4), we have some assumptions that \(\ln(1 + \gamma) \simeq \ln(\gamma)\) and the block fading channel during every single packet transmission. With our proposed selection criterion, the only selected best relay node amplifies its pilot and forwards to the both users in order to estimate the channel quality of each path for a suitable constellation size of adaptive modulation. Thus, we can reduce the system complexity. In order to prove the efficiency of our selection criterion, we will compare our proposed scheme to a much more complex scheme which searches exhaustively over all possible relay combinations to achieve the optimal overall spectral efficiency in our numerical results section.
C. Signal-to-Noise Ratio

Similar to the single relay case in (2), the signal-to-noise ratio (SNR) of $U_1 \rightarrow U_2$ path via the $i$th relay can be expressed as $\gamma_i = \frac{P_i \gamma_i}{P_i \gamma_i + P_r \gamma_r + P_{\gamma_i} \gamma_{\gamma_i} + P_{\gamma_r} \gamma_{\gamma_r}}$, where $P_i$ is the transmit power at the $i$th relay node. Similarly, the SNR of $U_2 \rightarrow U_1$ path via $i$th relay can be written as $\gamma'_i = \frac{P_i' \gamma_i'}{P_i' \gamma_i' + P_r \gamma_r + P_{\gamma_i} \gamma_{\gamma_i} + P_{\gamma_r} \gamma_{\gamma_r}}$.

The SNRs with the selected best relay node by (4) can be approximated by their upper bounds as $\Gamma_1 = \max \{\gamma_i\}$ and $\Gamma_2 = \max \{\gamma'_i\}$, respectively. These upper bounds are reasonable because

$$\max \{\gamma_i \cdot \gamma'_i\} \leq \Gamma_1 \cdot \Gamma_2. \quad (5)$$

IV. PERFORMANCE ANALYSIS

In this section, we analyze the performances of TWR-AM based on the average spectral efficiency and the average BER. For convenience, the subscript 1 and 2 indicate the $U_1 \rightarrow U_2$ and $U_2 \rightarrow U_1$ paths, respectively.

A. Average Spectral Efficiency

1) Single Relay Case: In the proposed TWR-AM, since each path of relaying can be independently treated for the decoding process, the total average spectral efficiency of TWR-AM can be calculated as the sum of the average spectral efficiencies of the two paths. Based on the discrete adaptive modulation, the average spectral efficiency of TWR-AM can be evaluated by using (2) and (3) as below

$$\eta = \sum_{j=1}^{N} \sum_{n=1}^{N} n a_j(n), \quad (6)$$

where the discrete rate $n = \{1, 2, \cdots, N\}$, $a_j(n)$ are the probability that $\gamma_j$ falls in the $n$th region. For instance, the probability $a_j(n)$ is given by

$$a_j(n) = \begin{cases} P_{\gamma_j} \left( \frac{n+1}{T_j} \right) - P_{\gamma_j} \left( \frac{n}{T_j} \right), & n = 1, 2, \cdots, N-1, \\ 1 - P_{\gamma_j} \left( \frac{N}{T_j} \right), & n = N, \end{cases} \quad (7)$$

where the CDF of $\gamma_j$, denoted as $P_{\gamma_j}(\cdot)$, can be evaluated from (16) in Appendix A. For example, $P_{\gamma_j}(\cdot)$ for $U_1 \rightarrow U_2$ path can be written by setting the parameters as $a = \frac{P_1 + P_2}{P_1 P_2}$, $b = \frac{1}{T_1}$, $T_1 = P_1 \gamma_{1,2}$ and $T_2 = P_1 \gamma_{1,r}$. Similarly, $P_{\gamma_j}(\cdot)$ for $U_2 \rightarrow U_1$ path can be simply evaluated by setting the parameters as $a = \frac{P_1' + P_2'}{P_1' P_2'}$, $b = \frac{1}{T_1}$, $T_1 = P_1' \gamma_{1,2}$ and $T_2 = P_1' \gamma_{1,r}$. The average spectral efficiency of D-AM over the Rayleigh fading channel is given in [12] as

$$\eta_D = N - \sum_{n=1}^{N} \left( 1 - e^{-\frac{\gamma_D}{(N_0 P_0)\gamma_D}} \right), \quad (8)$$

where $P_0$ and $\gamma_0$ are the transmit power and the average channel gain for D-AM, respectively.

2) Relay Selection Case: Similar to the conventional TWR-AM, the upper bound of the average spectral efficiency of TWR-AM with the best relay selection can be obtained by using the upper bounds of SNR per each path, $\Gamma_1$ and $\Gamma_2$, as

$$\eta \leq \eta^{UB} = \sum_{j=1}^{N} \sum_{n=1}^{N} n b_j(n), \quad (9)$$

where $b_j(n)$ are the probability that $\Gamma_j$ falls in the $n$th region and can be obtained by replacing $P_{\gamma_j}(\cdot)$ with $P_{\gamma_j'}(\cdot)$ in (7) where $P_{\gamma_j'}(x) = \prod_{i=1}^{L} P_{1}^L$. $P_{\gamma_j'}(x)$ is the individual CDF of $\gamma'_j$ obtained by replacing $P_{\gamma_j}$ with $P_{\gamma_j'}$ in $P_{\gamma_j}(x)$. In conclusion, we can arrive at the desired results of the upper bound for the average sum spectral efficiency of TWR-AM in the same manner of TWR-AM.

B. Average Bit Error Rate

The approximate BER expression of both rectangular and non-rectangular $M$-QAM for coherent modulations with two dimensional Gray encoding is given by [15, Eq. 9.31]

$$\text{BER}(n, \gamma) \approx \frac{4}{n} Q(\sqrt{\frac{3}{2} \gamma}), \quad n \geq 3. \quad (10)$$

where $n = \log_2 M$. On the other hand, the exact BER expression of BFSK is given by

$$\text{BER}(1, \gamma) = Q(\sqrt{2 \gamma}). \quad (11)$$

With the BER expressions given in (10) and (11), the switching thresholds for adaptive $M$-QAM can be calculated by solving the inverse approximate BER expressions.

1) Single Relay Case: For discrete rates of adaptive modulation, the average BER (ABER) can be computed as the ratio of the average number of bits in error over the total average number of transmitted bits [12] and there are two kinds of paths in TWR. Thus, the ABER can be written as

$$\text{ABER} = \frac{1}{\eta} \sum_{j,k \neq k} \sum_{n=1}^{N} n \text{BER}(n), \quad (12)$$

where

$$\text{BER}(n) = \int_{\gamma_0}^{\gamma_0^{n+1}} \text{BER}(n, \gamma) \left[ (1 - P_{\gamma_j} \left( \frac{n}{T_j} \right)) p_{\gamma_j}(\gamma) \right] d\gamma. \quad (13)$$

In (13), $\gamma_0^{n+1} = \infty$. The PDF of $\gamma_j$, $p_{\gamma_j}(\cdot)$, can be easily obtained with the help of (17). In addition, the PDF of $\gamma'_j$, $p_{\gamma'_j}(\cdot)$, is also obtained from (17) by setting $a = 1$, $b = 1$, $T_j = P_1 \gamma_{1,2}$ and $T_k = P_1 \gamma_{1,r}$ which is used when one path of TWR fails to meet the minimum constellation size.

2) Relay Selection Case: Since we evaluate the upper bound of SNRs for each path in TWR-AM with the best relay selection, the ABER can be approximately computed by using $\Gamma_1$ and $\Gamma_2$. In other words, the approximate ABER of the proposed scheme can be obtained by replacing $p_{\gamma_j}(\cdot)$, $p_{\gamma'_j}(\cdot)$ and $P_{\gamma_j}(\cdot)$ with $p_{\gamma_j}(\cdot)$, $p_{\gamma'_j}(\cdot)$ and $P_{\gamma_j}(\cdot)$, where the PDF


Fig. 3. Average spectral efficiency of TWR-AM with BER_{T_1} = 10^{-3} and BER_{T_2} = 10^{-5} versus the average SNR over the balanced hops.

V. NUMERICAL EXAMPLES

In this section, we evaluate the average spectral efficiency and the average BER of the proposed TWR-AM by Monte-Carlo simulations and compare them with our provided analyses. In addition, for adaptive modulation, we consider the maximum constellation size \( N \) is 8.

Fig. 4 illustrates the average spectral efficiency of the proposed TWR-AM with the target BERs, BER_{T_1} = 10^{-3} and BER_{T_2} = 10^{-5}. In this example, the average SNR is set as \( \Gamma \), where \( P = P_t = P_r \). We observe that, although each average spectral efficiency of TWR paths is less than one of OWR because of the interference effect from other side, the sum rates of the proposed TWR-AM provides a considerable gain. In addition, our analytical results using the convenient upper bound are in a good agreement with our simulation results.

Fig. 5 shows a comparison of the spectral efficiencies of TWR-AM and opportunistic relaying with adaptive modulation (OR-AM) for \( L = 4 \) in i.i.d. condition with BER_{T} = 10^{-3} and 10^{-5}, and their counterparts of Shannon’s channel capacity bound while varying the position of the relay node. We choose \( P = P_t = P_r = 100 \) W, \( \sigma^2 = 1 \) W/Hz and \( d_{1,1} = 1 \). As shown in Fig. 5, our proposed TWR-AM provides better performance of the spectral efficiency than OR-AM. However, note that because of the requirements of the link performance and the adopted discrete rates, there exists a certain gap between achievable spectral efficiency and capacity bound. Regarding the geometrical position of the relay node, we have an optimal performance when the relay nodes is placed in the middle of the two users, that is, \( 0.5d_{1,2} \) under the conditions of the linear relay network and i.i.d. channels.

Figs. 6 illustrates the average BER of our proposed TWR-AM with several target BERs for \( L = 4 \) cases. We consider the non-i.i.d. Rayleigh fading channels where the average SNRs are set as \( \gamma_1 = \gamma_1 \), \( \gamma_{1,1} = \gamma_1(1+0.1(i-1)) \) and \( \gamma_{1,2} = \gamma_1(10-0.1(i-1)) \). Our results show that the BER performance of system is satisfactory with respect to the target BERs, and that our analysis is in good agreement with the simulation results.

APPENDIX A

STATISTICAL OF THE SNR FOR DUAL-HOP AF RELAYING

In this Appendix, we introduce some results on the statistics of dual-hop with AF relaying which are related to MacDonald RV [16]. Given two exponential RVs as \( \gamma_1 \sim \text{Exp}(\beta_1) \) and \( \gamma_2 \sim \text{Exp}(\beta_2) \), we define the modified MacDonald r.v. \( \Gamma \) as 

\[
\frac{\gamma_1 \gamma_2}{a \gamma_1 + \gamma_2 + b}
\]

where \( a \) and \( b \) are constants. The CDF of \( \Gamma \) can be expressed as

\[
P_T(x) = Pr \left[ \frac{\gamma_1 \gamma_2}{a \gamma_1 + \gamma_2 + b} < x \right] = \int_0^x Pr \left[ \gamma_2 > \frac{(a y + b) x}{y - x} \right] f_{\gamma_1}(y) dy
\]
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